math304-05-s21 (Eppolito)

Quiz 1

Due: 6th April 2021

Instructions: Legibly complete each of the following on lined paper and submit on Gradescope. Collaboration and outside help (in any form) are forbidden.

1. Prove or disprove (with complete justification either way) that the given subset is a subspace. If it is a subspace, give a basis and compute its dimension.
   (a) $V = \{a + bx + cx^2 \in \mathcal{P}_2(\mathbb{R}) : a + b + c = 0, a - b = 0\}$
   (b) $V = \{a + bx + cx^2 \in \mathcal{P}_2(\mathbb{R}) : a + b = c^2\}$
   (c) $V = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathcal{M}_{2\times 2}(\mathbb{R}) : a + d = b + c\right\}$
   (d) $V = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathcal{M}_{2\times 2}(\mathbb{R}) : ad - bc = 0\right\}$

2. Prove or disprove (with complete justification either way) that the map is linear.
   (a) $L\colon \mathbb{R}^3 \to \mathcal{M}_{2\times 2}(\mathbb{R}), \quad L(x, y, z) = \begin{bmatrix} x - y & x + y^2 \\ z & x + y + z \end{bmatrix}$
   (b) $L\colon \mathcal{M}_{2\times 3}(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R}), \quad L\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix} = (a_{1,1} + a_{2,1})x + (a_{1,2} + a_{2,2})x^2 + (a_{1,3} + a_{2,3})x^3$

3. Compute bases for the kernel and range of the linear transformations given below.
   (a) $L\colon \mathbb{R}^3 \to \mathbb{R}^4, \quad L(x, y, z) = (x - y + z, x + y + z, -y, x - y - z)$
   (b) $L\colon \mathcal{P}_3(\mathbb{R}) \to \mathcal{M}_{2\times 2}(\mathbb{R}), \quad L(a + bx + cx^2 + dx^3) = \begin{bmatrix} a + b & b + c \\ c + d & a + d \end{bmatrix}$
   (c) $L\colon \mathcal{M}_{2\times 2}(\mathbb{R}) \to \mathbb{R}^3, \quad L\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a + b - c, a + d, b - c)$